#1 FOUR PIECES

Divide the shape below along the lines into 4 congruent pieces, each of 5 squares. Pieces may be rotated or reflected.

![Diagram of a shape divided into 4 pieces](image)

**Solution:**

![Solution diagram](image)

#2 TOGETHERNESS

The average age of a group of students and teachers is 20. The average age of the teachers is 35, while the average age of the students is 15. By what ratio are the teachers outnumbered?

**Solution:** If there are $S$ students and $T$ teachers, then the sum of the teachers' ages is $35T$ and the sum of the students' ages is $15S$, while the sum of all of their ages is $20(S+T)$.

Then $15S + 35T = 20(S+T)$,

so $15S + 35T = 20S + 20T$

and so $15T = 5S$,

giving $S = 3T$. Thus, the teachers are outnumbered 3 to 1.
#3 ROUND 'N ROUND 'N ROUND

Each of the three circles below has radius 6, and each is tangent to the others. What is the area of the white space between them?

Solution: Joining the centers gives an equilateral triangle 12 units on a side. Its altitude is $6\sqrt{3}$ and so its area is $36\sqrt{3}$. Each of the three wedges is a sixth of a circle and so has area $36\pi / 6 = 6\pi$. The white area is the triangle minus three of the wedges, $A = 36\sqrt{3} - 18\pi$.

#5 DOOR-STOPPER

AD is perpendicular to BC, AB is perpendicular to BD. BC is three times as long as AC, and triangle ABD has area 200. What is the area of triangle ABC?

Solution: Because angles ABC and BDC are complementary to A they are equal, and so the three right triangles ABC, BDC, ADB are similar. Then CD = 3BC = 3(3(AC)). Thus AD = 10 AC and so the area of ABD = 10(area of ABC) [they have the same altitude]. Thus the area of ABC is $200/10 = 20$. 
#4 CRY UNCLE

Jenni has three uncles, all from the Barhinkle family. Their names are Albert, Charles, and Nathan. They live in small towns called Appline, Currsen, and Nohau, in Arizona, California and Nevada. None of these towns is in a state with the same initial and no uncle's initial matches that of his state or town. If there is no Currsen in Arizona, where does Nathan Barhinkle live?

Solution: Currsen is not in Arizona and it is not in California, so it must be in Nevada; The uncle there must be Albert. Appline is not in Nevada and it is not in Arizona, so it is in California; The uncle there must be Nathan.

#6 LETTERS, NUMBERS

If H is 10, and T is half of M, how could MATH be 42, TEAM be 40 and MEET be 37?

Solution: 2 = MATH - Team = H - E and so E = 8. Also, 3 = TEAM - MEET = A-E and so A = 11. Then 3T = T + 2T = T+M = MEET - EE = 37 - 16 = 21, so T = 7 and thus M = 14.

#7 NO MORE THAN ONE

Find all real values of x for which \[ \frac{x - 3}{2x - 5} \leq 1. \]

Solution: Subtracting 1 from both sides gives

\[ 0 \geq \frac{x - 3}{2x - 5} - 1 = \frac{x - 3 - (2x - 5)}{2x - 5} = \frac{-x + 2}{2x - 5}. \] This last fraction is non-positive exactly when top and bottom have different signs (or the top is 0). Thus we have

(I) \( -x + 2 \leq 0 < 2x - 5 \) \ OR \ (II) \( 2x - 5 < 0 \leq -x + 2. \)

In case (I), \( x \geq 2 \) AND \( x > 5/2 \); These both imply \( x > 5/2 \).
In case (II), \( x < 5/2 \) AND \( x \leq 2 \); these imply that \( x \leq 2 \).
Thus our solution is \( x \leq 2 \) OR \( x > 5/2 \). In interval notation, the solution set is \(-\infty, 2\] U \((5/2, \infty)\).
**#8 SUM PAIRS**

The four numbers \(a < b < c < d\) can be paired in six different ways. If each pair has a different sum and the four smallest sums are 1, 2, 3, and 4, what are all of the possible values of \(d\)?

Solution: Because, \(a < b < c < d\), we have \(a+b < a+c < a+d < b+d < c+d\) and \(a+b < a+c < b+c < b+d < c+d\). The only pair we cannot compare is \(a+d\) and \(b+c\). One of them is 3 and the other is 4. Thus we have:

(I) \(a+b = 1, a+c = 2, a+d = 3, b+c = 4,\)

or

(II) \(a+b = 1, a+c = 2, b+c = 3, a+d = 4,\)

Solving (I) gives \(a = -1/2, b = 3/2, c = 5/2, d = 7/2\). Solving (II) gives \(a = 0, b = 1, c = 2, d = 4.\)

Thus, \(d\) must be \(7/2\) or \(4\).

**#9 GO FLY**

In the kite below, angles A and B have the same measure. What range of measures could A have, while preserving the kite shape?

Solution: Drawing the line AB divides the kite into two isoceles triangles. The base angles on top must be 50º, as shown above. The angle \(\alpha\) can be any acute angle, so \(A = 50º + \alpha\) is strictly between 50º and 140º.

**#10 IN ADDITION**

Consider the sum \(1! + 2! + 3! + 4! + \ldots + 2010!\) What is the units digit of this sum?

Solution: \(1! + 2! + 3! + 4! + 5! + 6! = 1 + 2 + 6 + 24 + 120 + 720 \ldots\) and every further factorial will be divisible by 10 and so end in 0. The last digit never changes after 4! and so it is 1+2 +6 + 4 = 3.